Math Curriculum

As the teacher, my role is to guide a discussion that helps students see that, for each circle they measure, the circumference is always about three times the diameter. It's important to point out to students that measurement is never exact, and even the best of measurements are approximations. That said, if we measure carefully with the best measuring tools we can find, for any circle, the result of dividing the circumference measurement by the diameter measurement will always be close to 3.14 or 3 1/7. That relationship—the ratio of a circle's circumference to its diameter—is what we call pi.

I've found interesting ways to assess my middle school students' understanding about circles. Although I want to know whether they've learned the relevant formulas, I also want to know whether they can apply that knowledge. One way to assess this is to ask them to solve the problem of measuring the diameter of a tree trunk. To help them envision the task, I model with a standard tape measure and a cylindrical container. The circumference of a container of bread crumbs, for example, measures about 12 1/4 inches, or 31 centimeters. (This is a good opportunity to reinforce for students that measurement is never exact.) After I ask students to predict the diameter of the container and explain their reasoning for

Then I presented another fraction and asked whether it was OK to cancel the zeros to produce an equivalent fraction:

Does 101/201 = 11/21?

Some students initially thought the answer was yes; others thought it was no. We had a spirited discussion. Trey argued for yes, because "It works for 102/204 and 12/24—both would be 1/2." Russell supported Trey with another example, 100/200 and 10/20, saying, "It doesn't matter which zeros you cancel."

Elissa argued that those examples were different because you could reduce both of them to 1/2, "but you can't reduce 101/201 or 11/21 to anything." Tina argued that the fractions should be the same because, "If you add 1 to each denominator, you get 101/202 and 11/22, and these are both equal to 1/2."

Sophia used a calculator to divide, and reported that it didn't work: 101 ÷ 201 was 0.5024875, and 11 ÷ 21 was 0.5238095. She came up to the board and recorded these numbers.

Then Nick came to the board and wrote the sequence of equivalent fractions he had written starting with 11/21 to show that 101/201 wasn't in the sequence:

11/21, 22/42, 33/63, 44/84, 55/105, 66/126, 77/147, 88/168, 99/189, 110/210

Emmy gave a place-value argument for why you can't cross out the middle zeros. She said, "If you cross out the zeros, you suddenly are making hundreds into tens, and math doesn't work like that."

Actually, there were three sides, with Leslie offering a minority opinion that the discussion was moot since both fractions were very, very close to 1/2, so you should just say that they're just about the same.

Too often, students learn rules without the depth of understanding that tells them when and when not to apply them. Here students have the opportunity to investigate what happens when "cancelling" to compare fractions, first with fractions for which cancelling maintains the proportional relationship between the numerators and denominators and then with fractions for which it doesn't. The question allows a variety of entry points for students to analyze what makes sense mathematically.

We should be mindful of what our students understand, not merely what they can do.

Those Who Understand, Teach

Learning how to best uncover the curriculum for students has been a long process for me. I've had to learn when to *ask* and when to *tell*. Even more important, I've had to learn *what* to ask and *what* to tell, which calls for thoroughly understanding the mathematical content I'm teaching.

Glenda Lappan, a past president of the National Council of Teachers of Mathematics, addressed the importance of teachers having deep content knowledge in her article "Knowing What We Teach and Teaching What We Know." She wrote:

> Our own content knowledge affects how we interpret the content goals we are expected to reach with our students. It affects the way we hear and respond to our students and their questions. It affects our ability to explain clearly and to ask good questions. It affects our ability to approach a mathematical idea flexibly with our students and to make connections. It affects our ability to push each student at that special moment when he or she is ready or curious. And it affects our ability to make those moments happen more often for our students.[3]

A friend of mine, also a math teacher, has a T-shirt with the following message: *Those who can, do. Those who understand, teach.* I agree with this message. Even with elementary math topics that seem fairly uncomplicated and easy to understand, unexpected twists and turns can emerge during classroom teaching. But if our math knowledge as teachers is robust enough, we can treat these surprises not as difficulties but as opportunities to guide students in uncovering their understanding of mathematics. EL

[1]I was first introduced to this idea by Annie Fetter's talk "Ever Wonder What They'd Notice?" at the National Council of Teachers of Mathematics conference in Indianapolis, Indiana, which can be viewed at www.youtube.com/watch?v=WFvYZDR4OeY.)

[2]National Governors Association for Best Practices & Council of Chief State School Officers. (2010). *Standards for Mathematical Practice.* Washington, DC: Authors. Retrieved from www.corestandards.org/Math/Practice.

[3]Lappan, G. (1999, November). Knowing what we teach and teaching what we know. *NCTM News Bulletin* (online newsletter). Retrieved from www.nctm.org/about/content.aspx?id=998.

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