Performance Assessment Task

Cindy's Cats
Grade 5

This task challenges a student to use knowledge of fractions to solve and multi-step problems with fractions. A student must show understanding of operations and fractions to find equivalent numerical representations to add and subtract fractions and compare size of fractions. A student must demonstrate an understanding of operations with fractions to multiply a fraction times a whole number.

Common Core State Standards Math – Content Standards

Number and Operations - Fractions

Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 1/4 = 8/12 + 3/12 = 11/12. (In general, a/b + c/d = (ad + bc)/bd.)

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < ½.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product (a/b) x q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a x q

5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to

3. Cindy's cats often share a carton of cat milk.

Sammy always drinks $\frac{1}{3}$ of the carton, Tommy always drinks $\frac{5}{12}$ of the carton, and Suzi always drinks $\frac{1}{6}$ of the carton.

What fraction of the carton of cat milk is left over? _____

Show how you figured it out.

4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.

Sammy used it for $\frac{1}{4}$ of the times and Tommy used it for $\frac{3}{10}$ of the times.

How many times did Suzi use the cat door? _____

Explain how you figured it out.

_____

_____

_____

_____

Task 3: Cindy's Cats

Cindy's Cats

Work the task and look at the rubric. What are the key mathematical ideas in this task?

When thinking about this task, students had access to the mathematics using a variety of strategies. Look at student work and see if you can categorize the strategies. Without judging whether the strategy was used with complete success, how many of your students attempted to use:

Common denominators	Decimals	Fractions over 100	Percents	Drawing or using a model	*Invented a strategy*

Now look at student work for part one. This is the entry level portion for most students.
How many of your students seemed to understand that the operation was addition?_____
How many of your students tried some kind of subtraction?_____
How many of your students have work where the operation is unclear?_____

Look at the answers in part 1. How many of your students have answers of:

7/8	87%	3/14	1/14	3/4	3/8	Whole number	Other

What do students understand? What are students confused about? In each case is the confusion caused by misunderstood or incorrectly memorized algorithms or procedures? Is the confusion a result of opportunity to learn? Is the confusion caused by not being able to think about the meaning of fractions? Is the confusion caused by not understanding operations?

Suzi	Either	Either	Each of
1/10	Whole number answer	3/10	them slept half a day

Many students understood that the operation involved in part 3 involved subtraction, but because they could only think about two fractions at a time they subtracted too early. What strategies do you use in class to help students figure out the correct operation for a problem? What kinds of models might help students make sense of the "action" of the problem?

Now look at student work for the final part of the task. What mathematics did students need to understand in order to work this task? What are the big mathematical ideas? What strategies do you think students have to make sense of this part of the task?

Look at student work. How many of your students put:

45	55	45/100 *or* 9/20	55/100 *or* 22/40	4/14	Other

What is the piece each student is missing? How are they showing a different misconception?

What do you think are some of the layers that need to be developed to build a deep understanding of fractions?
What are some of the issues that arise in working in context that don't show up when students work a page of practice problems?

Looking at Student Work on Cindy's Cats

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Student A, part 2

Student B, part 2

Student C is able to use diagrams to make sense of the size of the fractions and show the action or operation of the problems. In part 2, Student C shows the comparison in two different ways. *Can you describe the mathematics in each diagram?*

Student C

Student C, part 2

Student D uses percents to think about the situation in part 4.

Student D

In part 4, students had a difficult time interpreting their answers. Student E has done all the correct calculatioude can't break down th e meaning of the final answer from the number 45/100 to the meaning 45 times out of 100. *What kind of question could you pose to the class to get everyone thinking about what the 45/100 represents?*

Student E

Student F is able to use common denominators and goes the extra step to show how to find the denominators. Student F also struggles with understanding the meaning of the calculations. In part two the student is able to make the comparison by finding common denominators, and can see that there is a difference of one part. However, instead of thinking of the one as one part out of ten the

Student F, part 2

Student G has trouble choosing the operation in part 2 of the task. This is a comparison or subtraction operation and the student makes the common mistake of trying to add. In part 4 the student knows that subtraction is involved by doesn't have the correct whole, 100 times, to subtract from. The student also misses the idea of interpreting the answer from a fraction to number of times. *How do we help push students to develop the logical reasoning needed to interpret what is being asked versus what is being calculated?* This is important, because the nuances of logic needed to interpret the answer don't come up when students do practice exercises, but only arise through working tasks with rich contexts. *Do you think you give students enough opportunity to work and discuss mathematics in context?*

Student G

Student H has a total score of 3, but shows a lot of understanding about fractions. The student is

Student H, part 2

Student I also has a score of 3, but shows a very different profile of understanding. In part one, the student seems to want all fractions to be parts of 100 and actually makes correct calculations. However, the student tries to relate 87.5% back to a fraction and does that conversion incorrectly. In part 2 the student tries to find parts of the day by changing to a denominator of 24. The fraction conversions are messy and strategy breaks down in trying to deal with the remainders but could have yielded a correct solution. In part 3 the student again tries to make everything into parts of 100, but again has trouble dealing with the remainders. This strategy serves the student well in the final part of the task. *What might be some next steps for this student?*

Student I

Student I, part 2

Student J also tries to work with parts of 100 in part 1, but doesn't understand what to do with the remainder for 1/8. The student did not attempt other parts of the task. *What are some of the skills the student has that you can use to build further understanding?*

Student J

Student K works with the fractions in sets of two. Although the student has a score of 2, the student shows a great deal of understanding in some areas. In part one the student chooses the correct operation for the task, converts to an appropriate common denominator (actually working quite hard because each set requires a different denominator), and adds correctly. The student doesn't simplify the final answer correctly. Thinking about the problem in chunks makes it difficult to choose an operation in part three. While the overall action is subtracting or finding what's left over, this action doesn't happen until after the other quantities are totaled. Then the student needs to put in an implied quantity, the whole. *It would be interesting to interview a student with this type of error to see how he or she might solve a similar task involving just whole numbers. Would the student then see the addition? What do you think this student's personal algorithm is for finding denominators? Why does this make the task more difficult? What activities might help the student see the logic and simplicity of finding a smaller common denominator? What questions might you pose to the class?*

Student K

5th Grade Task 3 Cindy's Cats

Student Task	Solve fraction problems in a practical context. Use part/ whole relationships to solve problems.
Core Idea 2 Number Operations	**Understand the meanings of operations and how they relate to each other, make reasonable estimates and compute fluently.** • Reason about and solve problem situations that involve more than one operation in multi-step problems. • Use equivalent forms to add and subtract commonly used fractions. • Develop and use strategies to solve problems involving number operations with fractions relevant to students' experience.

Based on teacher observations, this is what fifth graders knew and were able to do:

- Find common denominators
- Convert to equivalent fractions

Areas of difficulty for fifth graders:

- Choosing operations in word problems
- Drawing accurate diagrams for fractions
- Interpreting what their calculations represent
- Understanding that fractions need to be converted to common denominators
- Choosing convenient common denominators (e.g. trying to switch to 24th's in part 2 was too difficult or interpreting remainders when trying to convert to 100th 's was difficult)

Strategies used by successful students:

- Using common denominators
- Labeling their work and their answers as they worked through the different steps
- Using percents
- Using accurate models (built with common denominators)

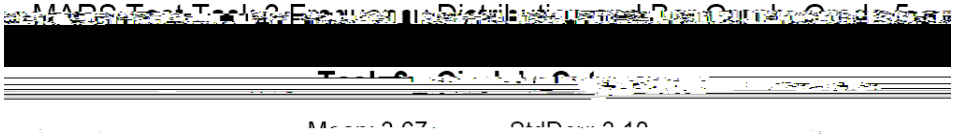

Table 27: Frequency Distribution of MARS Test Task 3, Grade 5

Task 3 Scores	Student Count	% at or below	% at or above
0	2842	36.7%	100.0%
1	492	36.0%	69.3%
2	668	43.2%	64.0%
3	455	49.1%	56.8%
4	665	55.3%	51.8%
5	615	61.9%	44.7%
6	1045	73.2%	38.1%
7	1101	85.1%	26.8%

Cindy's Cats

Points	Understandings	Misunderstandings
0	98% of the students with this score attempted the task.	Students did not understand how to work with common denominators. More than 12% added denominators to get 3/14 or 1/14. 5% added two of the three fractions to get 3/8 or 3/5. Other common errors for part 1 included 3/4, 1, 87%.
1	Most students with this score had a correct answer to part 1, but did not show enough of their work.	Students struggled with making the comparison.7% thought Suzi had 4/5 more and 3% thought Tommy had 4/5 more. 7% knew Suzi had the most but didn't quantify the difference and just wrote 7/10. 13% tried to find the number of hours (1 to 15 hours). Many students struggled with choosing the correct operation, adding instead of subtracting.
4	Students with this score could usually solve all of part 1 and 2, adding fractions with unlike denominators and doing comparison subtraction with f scn /TT1 1 Tf 12 0	

Implications for Instruction

Students need practice working with fractions in context. They should have a variety of strategies for combining fractions: models, common denominators, changing fractions to decimals or

Arnold

Berta

Carl

Ernie

Fanny

Fanny, part 2